

Measures of Linkage Disequilibrium

Part III: Relationship between D' and R

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This is the 3rd of three documents on two measures of linkage disequilibrium, D' and R . The measures D , D_{max} and D' are defined in Part I. R is defined in Part II. In this part we look at the relationship between the measures D' and R .

		Allele at 1st locus		
		A	a	
Allele at 2nd locus	B	p_{AB}	p_{aB}	p_B
	b	p_{Ab}	p_{ab}	p_b
		p_A	p_a	1

Table 1: Haplotype Probabilities

Assume that p_A , p_a , p_B and p_b in Table 1 are not equal to zero. D' and R are given by

$$R = \frac{D}{\sqrt{p_A p_a p_B p_b}} \quad \text{and}$$

$$D' = \frac{D}{D_{max}} .$$

From the above expressions, it follows that

$$R \times \sqrt{p_A p_a p_B p_b} = D' \times D_{max}$$

so

$$R = D' \times \frac{D_{max}}{\sqrt{p_A p_a p_B p_b}}.$$

In Part II, we defined D_{max} such that

$$D_{max} = \begin{cases} \min(p_A p_b, p_a p_B), & D \geq 0 \\ \min(p_A p_B, p_a p_b), & D < 0. \end{cases}$$

Thus, when $D \geq 0$ we have

$$R = \begin{cases} D' \times \sqrt{\frac{p_A p_b}{p_a p_B}} & \text{if } \min(p_A p_b, p_a p_B) = p_A p_b \\ D' \times \sqrt{\frac{p_a p_B}{p_A p_b}} & \text{if } \min(p_A p_b, p_a p_B) = p_a p_B. \end{cases}$$

When $D < 0$ we have

$$R = \begin{cases} D' \times \sqrt{\frac{p_A p_B}{p_a p_b}} & \text{if } \min(p_A p_B, p_a p_b) = p_A p_B \\ D' \times \sqrt{\frac{p_a p_b}{p_A p_B}} & \text{if } \min(p_A p_B, p_a p_b) = p_a p_b. \end{cases}$$

From the above, we can see that R is less than or equal to D' . For example, when $D \geq 0$ and $\min(p_A p_b, p_a p_B) = p_A p_b$ we have

$$R = D' \times \sqrt{\frac{p_A p_b}{p_a p_B}} < D'.$$